Exercice 1

1-) $f(x) = x^3 + (1/3)x$

$f'(x) = 3x^2 + (1/3)$

On $\mathbb{R}$; $f'(x) > 0$; implies that $f(x)$ is continuous and increasing. thus $f(x)$ has an inverse

- compute $(f^{-1})'(28)$

$$(f^{-1})'(28) = \frac{1}{f'(f^{-1}(28))}$$

$f^{-1}(28) = a \Rightarrow f(a) = 28$

$a^3 + (1/3)a = 28$

Exercice 2

- $\int x \sqrt[3]{x+1}\, dx$

$= \int x (x+1)^{\frac{1}{3}}\, dx$

$= \dfrac{(x+1)^{\frac{4}{3}}}{\frac{4}{3}} + C$

$\boxed{= \frac{3}{4} (x+1)^{\frac{4}{3}} + C}$

- $\int \cos x \sqrt{1-\cos^2 x}\, dx$

$= \sqrt{2} \int (2\cos^2 (x/2) - 1) \sin (x/2)\, dx$

let $u = x/2 \Rightarrow du/dx = 1/2 \Rightarrow dx = 2du$

$= \sqrt{2} \int (2\cos^2 (u) - 1) \sin(u) \cdot du$

let $v = \cos(u) \Rightarrow du = -1/\sin(u) \cdot dv$

$= -\sqrt{2} \int (2v^2 - 1)\, dv$

$=$

Exercice 2

1-)

- $\int \cos^4(x)\sin^4(x)$

$= \int \sin^4(x)\left[(1-\sin^2(x)\right]^2$

$= \int \sin^8(x).dx - 2\int \sin^6(x).dx + \int \sin^4(x).dx$

$= \left[\left(\frac{-\cos(x)\sin^7(x)}{8}\right) + \frac{7}{8}\int \sin^6(x)\,dx\right] - 2\int \sin^6(x)$

$+ \int \sin^4(x).dx$

$$\boxed{= \frac{\sin(8x) - 8\sin(4x) + 24x}{1024} + C}$$

- $\int \frac{x^2}{(x+2)}\,dx$

let $u = x+2 \rightarrow du/dx = 1 \rightarrow du = dx$

$= \int \frac{u-2}{u}\,du$

$= \int 1.du - 2\int \frac{1}{u}.du$

$= [u - (+2\ln(u)) + C]$

$$\boxed{= x+2 - 2\ln(x+2) + C}$$

- $\int 1/x^2 \cdot \sin^5\left(\frac{1}{x}\right) \cdot \cos^2 \frac{1}{x}\, dx$

let $u = \frac{1}{x} \rightarrow \frac{du}{dx} = -\frac{1}{x^2} \rightarrow dx = -x^2 du$

$= -\int \cos^2(u) \sin^5(u)\, du$

$= \int - \left(-\cos^2(u) \left(\cos^2(u) - 1\right)^2\right) \sin(u) \cdot du$

let $v = \cos(u)$; $dv/du = -\sin(u) \rightarrow$
$du = -1/\sin(u) \cdot dv$

$= + \int v^2 (v^2 - 1)^2\, dv$

$= \int (v^6 - 2v^4 + v^2)\, dv$

$= \int v^6 dv - 2\int v^4 dv + \int v^2 dv$

$= \frac{v^7}{7} - \frac{2v^5}{5} + \frac{v^3}{3}$

$= \frac{\cos^7(u)}{7} - \frac{2\cos^5(u)}{5} + \frac{\cos^3(u)}{3}$

$$\boxed{= \frac{\cos^7\left(\frac{1}{x}\right)}{7} - \frac{2\cos^5\left(\frac{1}{x}\right)}{5} + \frac{\cos^3\left(\frac{1}{x}\right)}{3} + C}$$

Exercice 2

2-)

$$\int_1^5 x\sqrt{2x-1}\,.dx$$

$$= \int_1^5 x\,(2x-1)^{\frac{1}{2}}\ dx$$

let $u = 2x-1 \rightarrow du/dx = 2 \rightarrow dx = 1/2\, du$

$$\frac{1}{4}\int_1^5 u+1\,.\,(u)^{\frac{1}{2}}$$

$$= \frac{1}{4}\int_1^5 u^{\frac{3}{2}} + u^{\frac{1}{2}}$$

$$= \frac{1}{4}\left[\int_1^5 u^{\frac{3}{2}} + \int_1^5 u^{\frac{1}{2}}\right.$$

$$= \frac{1}{4}\left[\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}\right]_1^5$$

$$= \left[\frac{(2x-1)^{\frac{5}{2}}}{10} + \frac{(2x-1)^{\frac{3}{2}}}{6} \right]_1^5$$

$$= \frac{9^{\frac{5}{2}}}{10} + \frac{9^{\frac{3}{2}}}{6} - \frac{1^{\frac{5}{2}}}{10} - \frac{1^{\frac{3}{2}}}{6}$$

$$\boxed{= \frac{9^{\frac{5}{2}} - 1}{10} + \frac{9^{\frac{3}{2}} - 1}{6}}$$

- $\int_0^1 \ln(1+2x).dx$

let $u = 2x+1 \rightarrow du/dx = 2 \rightarrow dx = 1/2\, du$

$$= \frac{1}{2} \int_0^1 \ln(u).du$$

$$= \frac{1}{2}\left(u\ln(u) - \int_0^1 1.du \right)$$

$$= \left[u\ln(u) - u \right]_0^1 \cdot \frac{1}{2}$$

$$= \frac{1}{2} \left[(2x+1)\ln(2x+1) - 2x+1 \right]_0^1$$

$$\boxed{= \frac{3}{2} \ln(2x+1) - \frac{3}{2} - (1/2)}$$